Pergamon

J. Appl. Maths Mechs, Vol. 61, No. 5, pp. 757–763, 1997

0021–8928/97 $24.00+0.00

PII: S0021–8928(97)00099-3

A CLASS OF POLYNOMIAL SOLUTIONS IN THE PROBLEM OF THE MOTION OF A GYROSTAT IN A MAGNETIC FIELD†

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(*Received* 10 *July* 1996)

The conditions for the existence of a class of polynomial solutions of the equations of motion of a gyrostat in a magnetic field are studied taking the Barnett–London effect [1, 2] into account. It is shown that if the condition of motion isoconicity is imposed when the moving and stationary hodographs of the angular velocity are symmetric images of one another in the plane tangent to them, the problem can be solved completely and yields two new cases of integrability of the equations of motion.

Polynomial solutions of the structure under consideration have been investigated completely in the classical problem of the motion of a rigid body [3–6] and partially in the generalized problem of dynamics, which can be described by the well-known hydrodynamical analogue of the Kirchhoff equation [7].

1. FORMULATION OF THE PROBLEM

It is well known that a rotating ferromagnetic medium becomes magnetized along the axis of rotation (the Barnett effect). The magnetic moment in $\mathbf{M} = B\boldsymbol{\omega}$, where B is a symmetric linear operator. A similar phenomenon also occurs in the case of a rotating rigid body (the London effect). Note that B is a diagonal operator in the principal system of coordinates of the gyrostat. We shall therefore consider equations of motion of the form [1, 2]

$$A_1\dot{\omega}_1 = (A_2 - A_3)\omega_2\omega_3 + \lambda_2\omega_3 - \lambda_3\omega_2 + B_2\omega_2\nu_3 - B_3\omega_3\nu_2 +$$
$$+s_2\nu_3 - s_3\nu_2 + (C_3 - C_2)\nu_2\nu_3, \quad \dot{\nu}_1 = \omega_3\nu_2 - \omega_2\nu_3 \quad (1\ 2\ 3) \tag{1.1}$$

They admit of only two first integrals

$$\sum_{i=1}^{3} (A_i\omega_i + \lambda_i)\nu_i = k, \quad \sum_{i=1}^{3} \nu_i^2 = 1 \tag{1.2}$$

In (1.1) and (1.2) ω_i are the components of the angular velocity vector, ν_i are the components of the magnetic field direction vector, λ_i are the components of the gyrostatic momentum, s_i are the components of the centre of mass, A_i are the components of the inertia tensor, B_i are the components of B, C_i are the components of the matrix C characterizing the Newtonian attraction, a dot over a variable denotes the corresponding derivative, and the symbol (123) indicates that the remaining equations can be obtained by cyclic permutation of indices.

In (1.1) and (1.2) we put

$$\omega_1 = p, \quad \omega_2 = q, \quad \omega_3 = r, \quad \lambda_1 = \lambda, \quad \lambda_2 = \lambda_3 = 0, \quad s_1 = s, \quad s_2 = s_3 = 0 \tag{1.3}$$

and we shall seek solutions of the resulting equations in the form

$$q^2 = Q(p) = \sum_{k=0}^{n} b_k p^k, \quad r^2 = R(p) = \sum_{i=0}^{m} c_i p^i, \quad \nu_1 = \varphi(p) = \sum_{j=0}^{l} a_j p^j \tag{1.4}$$

†*Prikl. Mat. Mekh.* Vol. 61, No. 5, pp. 781–787, 1997.

$$\nu_2 = q\psi(p),\quad \nu_3 = r\varkappa(p),\quad \psi(p)=\sum_{k=0}^{n_1} g_k p^k,\quad \varkappa(p)=\sum_{i=0}^{m_1} f_i p^i$$

Here $n, m, l, n_1, m_1 \in N$ or zero and b_k, c_i, a_j, g_k, f_i are constant parameters to be determined.

Note that in the classical problem of the motion of a rigid body the solutions of Goryachev [3], Steklov [4], and Kowalewsky [5] belong to these classes. For Steklov's solution $m = n = 2, l = 2, m_1 = n_1 = 1$, for Goryachev's solution $n = 2, m = 4, l = 4, n_1 = 3, m_1 = 1$, and for Kowalewsky's solution $n = 2, m = 3, l = 3, n_1 = 2, m_1 = 1$. Kharlamov [6] extended Steklov's and Kowalewsky's solutions to the case of gyrostat motion.

We substitute (1.4) into (1.1) and (1.2) and take (1.3) into account. We thereby obtain

$$A_1(\psi(p)-\varkappa(p))=\varphi'(p)[A_2-A_3+B_2\varkappa(p)-B_3\psi(p)+(C_3-C_2)\psi(p)\varkappa(p)] \tag{1.5}$$

$$A_2Q'(p)(\psi(p)-\varkappa(p))=2\varphi'(p)[(A_3-A_1)p+B_3\varphi(p)-B_1p\varkappa(p)- \tag{1.6}$$

$$-\lambda-s\varkappa(p)+(C_1-C_3)\varphi(p)\varkappa(p)]$$

$$A_3R'(p)(\psi(p)-\varkappa(p))=2\varphi'(p)[(A_1-A_2)p+B_1p\psi(p)-B_2\varphi(p)+ \tag{1.7}$$

$$+\lambda+s\psi(p)+(C_2-C_1)\varphi(p)\psi(p)]$$

$$(Q(p)\psi^2(p))'(\psi(p)-\varkappa(p))=2\varphi'(p)\psi(p)(p\varkappa(p)-\varphi(p)) \tag{1.8}$$

$$(R(p)\varkappa^2(p))'(\psi(p)-\varkappa(p))=2\varphi'(p)\varkappa(p)(\varphi(p)-p\psi(p)) \tag{1.9}$$

$$\dot p=(\varphi'(p))^{-1}(\psi(p)-\varkappa(p)(Q(p)R(p))^{1/2} \tag{1.10}$$

$$\varphi^2(p)-1+Q(p)\psi^2(p)+R(p)\varkappa^2(p)=0 \tag{1.11}$$

$$(A_1p+\lambda)\varphi(p)+A_2Q(p)\psi(p)+A_3R(p)\varkappa(p)=k \tag{1.12}$$

(the derivative with respect to p is denoted by a prime). Equation (1.10) can be used to determine $p(t)$.

Following [7], we shall assume that the gyrostat is subject to isoconic motion within the framework of (1.4), i.e. the relation

$$p(\varphi(p)-\varepsilon)+Q(p)\psi(p)+R(p)\varkappa(p)=0 \tag{1.13}$$

holds, where ε takes the values ± 1.

2. THE CASE $m_1 = n_1 = 0$

The estimation of the maximum degrees of polynomials is one of the main problems in the study of solutions (1.4). The solution of this problem gives rise to a number of singular cases.

Consider Eqs (1.5)–(1.7). Since $\psi(p) - \varkappa(p) \neq 0$, it follows that $\varphi(p)$ is a linear function, and $Q(p)$ and $R(p)$ are quadratic functions of p. Substituting these functions into (1.5)–(1.9) and (1.11), we obtain a system of algebraic equations, which imply, in particular, that $b_1g_0 + c_1f_0 = 0$, $a_0 = \varepsilon$, $b_0 = c_0 = 0$. We designate g_0 and f_0 to be the free parameters in the final solution of this system. Then we have

$$a_1=A_1(g_0-f_0)[A_2-A_3+B_2f_0-B_3g_0+(C_3-C_2)g_0f_0]^{-1}$$

$$b_2=a_1(f_0-a_1)g_0^{-1}(g_0-f_0)^{-1},\quad b_1=-2a_0a_1g_0^{-1}(g_0-f_0)^{-1}$$
$$c_2=a_1(a_1-g_0)f_0^{-1}(g_0-f_0)^{-1},\quad c_1=2a_0a_1f_0^{-1}(g_0-f_0)^{-1} \tag{2.1}$$

$$g_0(\lambda+sf_0)=a_0[(C_1-C_3)g_0f_0+B_3g_0+A_2]$$
$$f_0(\lambda+sg_0)=a_0[(C_1-C_2)g_0f_0+B_2f_0+A_3] \tag{2.2}$$

$$[A_2f_0+(A_1-A_3)g_0+B_1g_0f_0][A_2-A_3+B_2f_0-B_3g_0+(C_3-C_2)g_0f_0]-$$
$$-A_1(g_0-f_0)[A_2+B_3g_0+(C_1-C_3)g_0f_0]=0 \tag{2.3}$$

$$[A_3g_0+(A_1-A_2)f_0+B_1g_0f_0][A_2-A_3+B_2f_0-B_3g_0+(C_1-C_3)g_0f_0]-$$
$$-A_1(g_0-f_0)[A_3+B_2f_0+(C_1-C_2)g_0f_0]=0 \tag{2.4}$$

Assuming that $g_0(A_1-A_3)+f_0(A_2-A_1)\neq 0$, we can find C_1–C_3 and C_3–C_2 from (2.3) and (2.4). We can then find λ and s from (2.2), since $g_0-f_0\neq 0$. Equalities (2.1) give the values of the coefficients of the polynomial solution

$$q^2=p(b_2p+b_1),\quad r^2=p(c_2p+c_1),\quad \nu_1=a_1p+a_0$$
$$\nu_2=g_0[p(b_2p+b_1)]^{1/2},\quad \nu_3=f_0[p(c_2p+c_1)]^{1/2} \tag{2.5}$$
$$\dot p=a_1^{-1}(g_0-f_0)p[(b_2p+b_1)(c_2p+c_1)]^{1/2}$$

From (2.5) it follows that the solution can be expressed in terms of elementary functions of time.

3. THE CASE $n_1\neq 0$, $m_1=0$

We represent Eqs (1.11)–(1.13) in the form

$$Q(p)\psi(p)(A_2-A_3)=(A_3-A_1)p\varphi(p)-\lambda\varphi(p)-A_3\varepsilon p+k$$
$$R(p)\varkappa(p)(A_2-A_3)=(A_1-A_2)p\varphi(p)+\lambda\varphi(p)+A_2\varepsilon p-k \tag{3.1}$$

$$(A_2-A_3)(\varphi^2(p)-1)+\psi(p)[(A_3-A_1)p\varphi(p)-\lambda\varphi(p)-A_3\varepsilon p+k]+$$
$$+\varkappa(p)[(A_1-A_2)p\varphi(p)+\lambda\varphi(p)+A_2\varepsilon p-k]=0 \tag{3.2}$$

Two alternatives follow from (1.5)

$$n_1=l-1,\quad \text{if}\quad (C_3-C_2)f_0-B_3=0 \tag{3.3}$$
$$l=1,\quad \text{if}\quad (C_3-C_2)f_0-B_3\neq 0 \tag{3.4}$$

In the case (3.3) it is clear that $l>1$. We write Eqs (1.8) and (1.9) in the form

$$(Q(p)\psi^2(p))'(\psi(p)-f_0)=2\varphi'(p)\psi(p)(pf_0-\varphi(p))$$
$$R'(p)(\psi(p)-f_0)f_0=2\varphi'(p)(\varphi(p)-p\psi(p)) \tag{3.5}$$

If we assume that condition (3.4) is satisfied, it follows from (3.5) that $2n_1+n\leqslant 2$, which is impossible, since $n_1\neq 0$. It follows that only (3.3) is possible. Then equalities (3.5) yield $n=2$, $m\leqslant l+1$. Now consider (3.1) and (3.2). Given that $A_2=A_3$, these equalities imply that $l=1$. Therefore $A_2\neq A_3$, and from (1.6) and (1.7) we obtain

$$C_1=C_2,\quad B_3=0 \tag{3.6}$$

By (3.3) and (3.6) $C_1=C_2=C_3$, and these parameters do not appear in (1.1).
Consider the case $m=l+1$. It follows from (1.5)–(1.7), (3.1), (3.2) and (3.5) that

$$g_{l-1}=\mu la_l,\quad A_2\mu b_2=A_3-A_1-B_1f_0$$
$$A_3\mu(l+1)c_{l+1}=2(B_1g_{l-1}-B_2a_l),\quad b_2g_{l-1}(A_2-A_3)=(A_3-A_1)a_l$$
$$c_{l+1}f_0(A_2-A_3)=a_l(A_1-A_2),\quad (A_2-A_3)a_l+(A_3-A_1)g_{l-1}=0$$

$$b_2 g_{l-1}^2 + a_l^2 = 0, \quad (l+1)c_{l+1} f_0 g_{l-1} = 2la_l(a_l - g_{l-1}) \tag{3.7}$$
$$\mu = A_1^{-1}(A_2 - A_3 + B_2 f_0)$$

From (3.7), by eliminating g_{l-1}, c_{l+1} in the last equation we obtain $l = 1$, which is impossible. Therefore $m < l + 1$. Then by (3.7) we have

$$A_1 = A_2, \quad b_2 = -1, \quad \mu = 1/l, \quad g_{l-1} = a_l, \quad B_1 = B_2 \tag{3.8}$$

Let us consider the second equality in (3.1). Two alternatives follow from it: (1) $\lambda \neq 0, m = l$ or (2) $\lambda = 0, m = 1$. For the second alternative the second equation in (3.5) implies that $a_l = g_{l-1}, g_{l-2} = a_{l-1}$, since $n_1 = l - 1$. But because of (3.8), Eq. (1.6) yields the equality $g_{l-2} = (l-1)l^{-1}a_{l-1}$, which together with the equalities obtained before leads to a contradiction.

Consider the case when $\lambda \neq 0, m = l$. By (1.5) $\psi(p) = f_0 + \mu\varphi'(p)$. Let us compare the expressions for $R(p)$ obtained from (3.1) and (3.5). We then have an equation from which to determine $\varphi(p)$

$$\mu(\lambda^* + p)\varphi'(p) - \varphi(p) + pf_0 + \alpha^*\mu = 0 \tag{3.9}$$
$$\lambda^* = \lambda[2(A_1 - A_3)]^{-1}, \quad \alpha^* = A_1\varepsilon[2(A_1 - A_3)]^{-1}$$

We now consider Eqs (3.2) and (3.9), assuming that $l > 2$. Equating the coefficients of the powers $2l - 1$ and $l - 1$, we obtain $2l\lambda^* a_l - a_{l-1} = 0$ and $l\lambda^* a_l - a_{l-1} = 0$, respectively, which is impossible. It follows that $l = 2$, and from (3.2) and (3.9) we again have the relations

$$4\lambda^* a_2 - a_1 + 2f_0 = 0, \quad 2\lambda^* a_2 - a_1 + 2f_0 = 0$$

which cannot be satisfied for $a_2 \neq 0$. It follows that the case $n_1 \neq 0, m_1 = 0$ is impossible.

4. THE CASE $l = 1$

This case is also singular. We assume that $A_2 \neq A_3$. Then (3.1) implies that $m = n = 1, m_1 = n_1 = 1$. Consider the equation obtained as a result of adding (1.8) and (1.9). Equating to zero the coefficient of p in this equation, we obtain

$$3(b_1 g_1 + c_1 f_1) = -2a_1 \tag{4.1}$$

The isoconicity condition (1.13) yields $b_1 g_1 + c_1 f_1 = -a_1$, which contradicts (4.1). Consequently, one cannot put $A_2 = A_3$. If we assume that $n_1 > m_1$, then the condition for the degrees of the polynomials in (1.9) and (1.10) to be the same implies that $n + n_1 = 2, m + m_1 = 2$, i.e. we again arrive at the case considered above. When $m_1 = n_1$, the same equations lead to the conditions $n + n_1 \leqslant 2, m + n_1 \leqslant 2$, i.e. no new cases will appear. The case when $l = 1$ is impossible, in general.

5. THE GENERAL CASE

Consider the case when $l \neq 1, n_1 \neq 0, m_1 \neq 0$. In this case (3.1) implies that $A_2 \neq A_3$. The maximum degree of the polynomial on the right-hand side of (1.5) is attained by the term containing the product of the function and hence $\varphi(p)$, $\varkappa(p)$, and hence it is necessary to put

$$C_3 = C_2 \tag{5.1}$$

Let $A_1 \neq A_2, A_3 \neq A_1$ (the case of equality leads to $m = n = 2, m_1 = n_1 = l - 1$). Then from (3.1) we have $n + n_1 = l + 1, m + m_1 = l + 1$. We now consider (3.2) and assume that $n_1 > m_1$. Then $n_1 = l - 1$, and so $n = 2$. Since $m_1 < n_1 < l - 1$, from (1.5) we obtain $B_3 = B_2 = 0$ and

$$\psi(p) - \varkappa(p) = \mu_*\varphi'(p), \quad \mu_* = (A_2 - A_3)A_1^{-1}$$

Using this equality we can eliminate the difference $\psi(p) - \varkappa(p)$ in (1.6). As a result, we obtain an equation whose left-hand side is a polynomial of degree one and the right-hand side contains the expression $(C_1 - C_2)\varphi(p)\varkappa(p)$, i.e. we must put $C_2 = C_1$. Analysing the equation in question in a similar way, we get $B_1 = 0$. The whole set of conditions which has been established gives the classical problem [6].

Let $m_1 = n_1$. Then $m = n$, $n + n_1 = l + 1$. From (1.5)–(1.7) it follows that $B_2 \neq B_3$, $n_1 = l - 1$, $n = m = 2$ and

$$f_{n_1} = \mu_1 g_{n_1}, \quad f_{n_1 - 1} = \mu_1 g_{n_1 - 1}, \; \ldots, \; f_1 = \mu_1 g_1$$

$$a_l = \frac{\mu_2}{l\mu_3} g_{l-1}, \quad a_{l-1} = \frac{\mu_2}{(l-1)\mu_3} g_{l-2}, \; \ldots, \; a_2 = \frac{\mu_2}{2\mu_3} g_1, \quad a_1 = \frac{g_0 - f_0}{\mu_3} \tag{5.2}$$

$$\mu_1 = \frac{B_3}{B_2}, \quad \mu_2 = (B_2 - B_3)B_2^{-1}, \quad \mu_3 = (A_2 - A_3 + B_2 f_0 - B_3 g_0)A_1^{-1}$$

We consider equalities (3.1), (3.2) and (5.2), from which to determine b_2, c_2 and the conditions imposed on the parameters. We have

$$b_2 = \frac{\mu_2(A_3 - A_1)}{l\mu_3(A_2 - A_3)}, \quad c_2 = \frac{\mu_2(A_1 - A_2)}{l\mu_3(A_2 - A_3)}$$
$$\mu_2(A_2 - A_3)l^{-1}\mu_3^{-1} + A_3 - A_1 + \mu_1(A_1 - A_2) = 0 \tag{5.3}$$

Eliminating the difference $\psi(p) - \varkappa(p)$ in (1.6) and (1.7) using (1.5), we obtain

$$A_2\mu_3(2b_2 p + \ldots) = 2g_{l-1}(B_3\mu_3^{-1}l^{-1} - B_1\mu_1)p^l + \ldots$$
$$A_3\mu_3(2c_2 p + \ldots) = 2g_{l-1}(B_1 - B_2\mu_2\mu_3^{-1}l^{-1})p^l + \ldots \tag{5.4}$$

Since $l > 1$, from (5.3) and (5.4) it follows that

$$\mu_3 l = B_3\mu_2 B_1^{-1}\mu_1^{-1} = (B_2 - B_3)B_1^{-1} \tag{5.5}$$

Comparing (5.3) and (5.5), we obtain the condition

$$A_1(B_2 - B_3) + A_2(B_3 - B_1) + A_3(B_1 - B_2) = 0 \tag{5.6}$$

which can be parametrized as follows:

$$A_i = \varkappa_0 + \varkappa_1 B_i \tag{5.7}$$

On the basis of (5.3), (5.5) and (5.7) we have

$$b_2 = \frac{B_1(B_3 - B_1)}{B_2(B_2 - B_3)}, \quad c_2 = \frac{B_1(B_1 - B_2)}{B_3(B_2 - B_3)} \tag{5.8}$$

We assume that $l > 2$. Then the expanded expression for the right-hand side of (5.4) yields

$$B_2 a_{l-1} - B_1 g_{l-2} - s g_{l-1} = 0 \tag{5.9}$$

By (3.2) it follows from (5.9) that

$$g_{l-2} = \frac{s(l-1)}{B_1} g_{l-1} \tag{5.10}$$

We equate to zero the coefficients of p^{l-1} in (1.13). Then

$$a_{l-1}+g_{l-1}(b_1+\mu_1 c_1)+g_{l-2}(b_2+\mu_1 c_2)=0 \tag{5.11}$$

But $b_2 + \mu_2 c_2 = -B_1 B_2^{-1}$ by (5.3) and (5.8). Then from (5.11) we have

$$b_1+\mu_1 c_1=-s/B_2 \tag{5.12}$$

If we consider the equation obtained as a result of adding (1.8) and (1.9) and eliminate $\psi(p) - \varkappa(p)$ in it using (1.5), then equating to zero the coefficient of p^{l-1} we obtain

$$2(b_2+\mu_1 c_2)(l-1)g_{l-2}+(b_1+\mu_1 c_1)(2l-1)g_{l-1}+2(l-1)a_{l-1}=0$$

However, this equality contradicts (5.8)–(5.12), so $l = 2$.

Thus, in the general case we have

$$\begin{aligned}
&q^2=Q(p)=b_2p^2+b_1p+b_0, \quad r^2=R(p)=c_2p^2+c_1p+c_0\\
&\nu_1=\varphi(p)=a_2p^2+a_1p+a_0, \quad \nu_2=q(g_1p+g_0), \quad \nu_3=r(f_1p+f_0)\\
&\dot p=\frac{\alpha-\beta}{2}(Q(p)R(p))^{1/2}, \quad \alpha=\frac{B_2}{B_1}, \quad \beta=\frac{B_3}{B_1}
\end{aligned} \tag{5.13}$$

Relations (5.13) are a solution of (1.5)–(1.13) subject to the conditions

$$\begin{aligned}
&\beta^2+\beta(\alpha-1)+\alpha(\alpha-1)=0, \quad 3\varkappa_0=\varkappa_1 B_1\\
&A_1=\frac{4\varkappa_1 B_1}{3}, \quad A_2=\frac{\varkappa_1(B_1+3B_2)}{3}, \quad A_3=\frac{\varkappa_1(B_1+3B_3)}{3}\\
&s=-\frac{3\varepsilon\alpha\beta B_1}{\varkappa_1(\alpha+\beta)}, \quad \lambda=\frac{2}{3}s\varkappa_1\\
&a_2=\frac{\varkappa_1 B_1(1-\alpha)(1-\beta)}{2s\alpha\beta}, \quad b_2=-\frac{(1-\beta)}{\alpha(\alpha-\beta)}, \quad c_2=\frac{1-\alpha}{\beta(\alpha-\beta)}\\
&a_1=\frac{2\varkappa_1(1-\alpha-\beta)}{3\alpha\beta}, \quad b_1=-\frac{4s}{3\alpha B_1(\alpha-\beta)}, \quad c_1=\frac{4s}{3\beta B_1(\alpha-\beta)}\\
&g_1=\frac{\varkappa_1 B_1(1-\alpha)(1-\beta)}{2s\beta}, \quad f_1=\frac{\varkappa_1 B_1(1-\alpha)(1-\beta)}{2s\alpha}\\
&a_0=\frac{s\varkappa_1(2-3\alpha-3\beta)}{9\alpha\beta B_1}, \quad b_0=\frac{4s^2(1-2\alpha-\beta)}{9\alpha B_1^2(\beta-\alpha)(1-\alpha)(1-\beta)}\\
&c_0=\frac{4s^2(1-\alpha-2\beta)}{9\beta B_1^2(\alpha-\beta)(1-\alpha)(1-\beta)}, \quad g_0=\frac{\varkappa_1(1-\alpha-2\beta)}{3\beta}, \quad f_0=\frac{\varkappa_1(1-\beta-2\alpha)}{3\alpha}
\end{aligned}$$

We shall give an example when the solution is real-valued. Let $\alpha = 1/2$. Then $\beta \approx 0.8$ and there is a range of ps in which the conditions $Q(p) \geqslant 0$, $R(p) \geqslant 0$ are satisfied simultaneously. Assuming that $\varkappa_1 > 0$, it can be shown that the restrictions on the moments of inertia are also satisfied. Solution (5.13) can be represented in the form of Jacobi elliptic functions.

Thus, it has been proved that isoconic motions within the framework of polynomial solutions exist only in two cases: (1) $n_1 = m_1 = 0$ and (2) $n = m = l = 2$, $n_1 = m_1 = 1$, which give two new solutions (2.5) and (5.13) of (1.1).

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Translated by T.J.Z.